**Increased resistance to buckling of piston rods through induction hardening**

**Summary**
Induction hardening of hydraulic cylinder piston rods engenders not only a protection from damage in the event of external impact but also improves the mechanical properties of the rod and in particular, the resistance to buckling failure in push mode. This increase in buckling resistance can be accounted for by considering the rod as a composite material with a tube of hardened steel surrounding a solid core. Consequently, in relation to a non-hardened rod with the same steel base, the diameter of an induction-hardened rod can be downsized leading to benefits in the form of reduced weight and lower costs.

**List of symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$D$</td>
<td>rod diameter or O.D. of tube (mm)</td>
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<tr>
<td>$L$</td>
<td>unsupported rod length (mm)</td>
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<tr>
<td>$\lambda$</td>
<td>slenderness parameter</td>
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<td>$F_b$</td>
<td>buckling force (kN)</td>
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<td>$\alpha, \Phi$</td>
<td>dimensionless parameters</td>
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<td>$E$</td>
<td>modulus of elasticity (N/mm$^2$)</td>
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<td>$\sigma_y$</td>
<td>compressive yield stress (N/mm$^2$)</td>
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<tr>
<td>$\sigma_b$</td>
<td>buckling stress (N/mm$^2$)</td>
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<tr>
<td>$\sigma_{be}$</td>
<td>Euler buckling stress (N/mm$^2$)</td>
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<tr>
<td>$d$</td>
<td>I.D. of tube (mm)</td>
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Induction hardening of piston rods in hydraulic cylinders is performed in order to increase the resistance to damage from external impact in applications where there is risk for such; examples are piston rods for hydraulic cylinders in excavators and loaders and for power-steering cylinders. An additional advantage afforded by induction hardening is improved resistance to inadvertent handling damage during cylinder manufacture.

However, induction hardening also alters the mechanical characteristics of the steel in the piston rod, a fact which is normally not recognised by engineers involved in design and dimensioning of hydraulic cylinders. A simple demonstration of such changes is afforded by Fig.1 in which stress-strain curves from tensile testing of full-section hard-chrome plated bars in both hardened and non-hardened condition are compared (280X steel base - 20MnV6 improved). The induction-hardened rod is characterised by higher yield strength, higher rate of strain hardening, considerably increased tensile strength and somewhat reduced elongation in comparison with the non-hardened material. These effects can reasonably well be accounted for using a simple law of mixtures based on the different properties of the steel in case and core.
Fig. 1 Stress-strain curves in tension for full section hard-chrome plated bar comparing hardened and non-hardened execution (280X grade, diameter 30 mm).

A further striking example of the influence of induction hardening on the mechanical characteristics of piston rods is afforded by bend testing. In Fig. 2, stress-strain curves for full-section hard-chrome plated bars in 3-point bending are compared for hardened and non-hardened bars of two different diameters, 30 and 60 mm (again, the base steel grade is 280X).

Fig. 2 Stress-strain curves in 3-point bending comparing hardened and non-hardened hard-chrome plated piston-rod material (base steel 280X).
In bending, the maximum stress is experienced in the outer fibre and it is therefore logical that the resistance to deformation is markedly improved as a consequence of the presence of the induction-hardened case. Furthermore, the resistance to bending is greater for the 30 mm than for the 60 mm bar; this is because the hardened zone occupies a greater proportion of the bar section for the smaller diameter (the hardness distributions in the induction-hardened case are shown in Fig. 3). Note also that the stress-strain curves in bending are virtually independent of dimension for standard, i.e. non-hardened, rods.

![Hardness distribution in induction-hardened, hard-chrome plated rods giving the 3-point bending data in Fig. 2 (base steel grade 280X).](image)

**Induction hardening also improves buckling resistance**

Since buckling is essentially a phenomenon involving bending under axial compressive stress, it is pertinent to enquire whether the pronounced influence of induction hardening on bend strength, as demonstrated in Fig. 2, is reflected in a similar improvement in the resistance to buckling. In Euler’s classical treatment, the stress required for buckling is only governed by a single mechanical characteristic, namely E-modulus; in particular the Euler buckling stress does not depend on typical characteristics of plastic deformation such as yield stress or ultimate tensile stress. However, it is well known that for small values of slenderness parameter, Euler’s equation is no longer valid and the buckling resistance does show a dependence on yield and tensile strength.

We have compared the resistance to buckling of hard-chrome plated bars (280X base steel) with different diameters and lengths and in non-hardened and induction-hardened execution. The testing was performed in an instrumented hydraulic press, which was adapted for the purpose. The test data were monitored in the form of a force-time curve. Two such curves for hardened and non-hardened rods with diameter \(D\) 44.45 mm and length \(L\) 650 mm (slenderness parameter, \(\lambda=4.5\)) are reproduced in Fig. 4.
Fig. 4 Comparison of force-time curves in buckling tests on hard-chrome plated non-hardened and induction hardened pistons rods (grade 280X).

Buckling occurs at the peak of the force-time curve and it is quite clear that the buckling force for the hardened rod is 22% higher than that for the standard rod. It would seem that induction hardening does indeed have a positive effect in terms of resistance to buckling.

By testing hardened and non-hardened rods with different diameters and lengths, the buckling stresses have been established for a range of slenderness parameters from 50 to 150. The rod diameters tested were 30 and 44.45 mm; unfortunately, limitations with the equipment precluded testing of larger diameters. The results from all tests are shown in Fig. 5 as a buckling diagram, i.e. buckling stress vs. slenderness parameter ($\lambda$). It is evident that the buckling stress is increased as a result of induction hardening over the entire range of slenderness parameter investigated. There is some scatter in the data, probably deriving from the makeshift testing equipment, but the effect of hardening is unequivocal. The improvement engendered by induction hardening is about 20% at $\lambda=50$, 30% at $\lambda=100$ and 60% at $\lambda=150$. In other words, it appears that induction hardening improves buckling resistance even in the elastic regime at high slenderness parameters, where the Euler buckling stress is normally considered accurate; indeed, the influence of hardening is proportionally even more significant in the high-$\lambda$, elastic regime.
Can the influence of induction hardening on resistance to buckling be explained?

The observations summarised in Fig. 5 can simple-mindedly be explained if the induction hardened rod is treated as a composite material having a hardened case in the form of a tube which is filled with a solid core of base steel. Considering first a solid homogeneous rod, the relationship between buckling stress and slenderness parameter for buckling of columns is defined in a number of building standards as indicated below. These relationships can also be applied to piston rods subjected to compressive (push) loading.

We define a dimensionless parameter $\alpha$ as:

$$\alpha = \frac{1}{\pi} \frac{\lambda}{\sqrt{\frac{E}{\sigma_s}}}$$

where $E$ is the modulus of elasticity, $\sigma_s$ the yield stress in compression and $\lambda = 4L/D$ for a solid cylinder.

The buckling stress, $\sigma_b$, is then given by

$$\sigma_b = \sigma_s \left( \phi - \sqrt{\phi^2 - \frac{1}{\alpha^2}} \right)$$

in which the factor $\Phi$ is defined as:

$$\phi = 1 + 0.21(\alpha - 0.2) + \alpha^2$$

For large $\lambda$ (long, smaller diameter rods), $\sigma_b$ calculated from the above is very close to, but always slightly less than the Euler buckling stress, $\sigma_{be}$, for a rod with end fixtures which are free to rotate about an axis orthogonal to that of the rod (Euler Case II), i.e.

$$\sigma_{be} = \pi^2 \frac{E}{\lambda^2}$$

where obviously $\frac{1}{\alpha^2} = \frac{\sigma_{be}}{\sigma_s}$.
As $\lambda \to 0$, the above expressions give $\sigma_b > \sigma_s$ which is not feasible and $\sigma_s$ is then set equal to the yield stress in compression. The curve calculated with the help of the above equations for grade 280X ($\sigma_s = 590$ N/mm$^2$, $E = 210,000$ N/mm$^2$) is shown in Fig. 5 above. The agreement with the measured data for non-hardened rod is reasonable.

In order to explain the observations on induction-hardened rods (see Fig. 5), it is assumed that the buckling stress is the same as for a tube with a wall thickness equal to the thickness of the induction-hardened case. The slenderness parameter for a tubular piston rod with O.D. = $D$ and I.D. = $d$ is

$$\lambda_{\text{tube}} = 4L \sqrt{\frac{D^2 - d^2}{D^4 - d^2}}.$$

If the case thickness is 1.5 mm (see Fig. 3), then $d = D - 3$ mm and the slenderness parameter for the “tube” of induction-hardened steel can be evaluated. The corresponding value of $\sigma_s$ can then be evaluated from the above relationships and plotted against the true $\lambda$ for the solid bar. The curve obtained in this way is plotted in Fig. 5 and it can be seen to conform quite well to the observed buckling behaviour for the induction hardened bar lending support to the contention that in terms of buckling, the induction-hardened rod behaves like a tube with a core of softer material.

**Practical consequences in relation to dimensioning of piston rods**

The present study has clearly demonstrated that induction hardening of piston rods improves not only the resistance to external impact but also the mechanical characteristics of the rod. In particular, the resistance to buckling failure is increased markedly. The implication is that an induction hardened rod can be downsized significantly in relation to a standard, non-hardened rod whilst still maintaining an acceptable safety factor in relation to buckling failure. This is illustrated by the following examples:

**Example 1** Cylinder with bore 100 mm and maximum operating pressure 260 bar.

* A non-hardened rod, diameter 50 mm and length 1 000 mm ($\lambda = 80$) in steel grade 280X (min. yield strength 520 N/mm$^2$), has a calculated buckling stress of 262 N/mm$^2$; the maximum stress in the rod is 104 N/mm$^2$ and the safety factor against buckling failure is 2.52.

* For an induction-hardened rod, diameter 45 mm and length 1 000 mm ($\lambda = 88.9$) in the same steel grade, the buckling stress calculated with the above model is 353 N/mm$^2$; the maximum stress in the rod is now 128 N/mm$^2$, and the safety factor is 2.76. In other words, the smaller diameter induction-hardened rod actually offers a greater margin of safety against buckling.

**Example 2** Cylinder with bore 80 mm and maximum operating pressure 140 bar.

* For a non-hardened rod, diameter 40 mm and length 1 500 mm ($\lambda = 150$), the calculated buckling stress is 85 N/mm$^2$ and since the maximum stress is the rod is 56 N/mm$^2$, the safety factor is 1.52.

* An induction hardened rod, diameter 35 mm and length 1 500 mm ($\lambda = 171.4$) has a buckling stress of 117 N/mm$^2$ and the maximum stress in the rod is now 73 N/mm$^2$ giving a safety factor of 1.60.

The weight saving gained by switching to the induction-hardened rod in the above examples is 19% and 23% respectively and in addition, the extra cost of induction hardening is normally more than compensated for by the lower weight of the rod. A further advantage afforded by the downsized induction-hardened rod in relevant applications is a greater pull force because of the increased annular area.